The Taming of the Skew: Asymmetric Inflation Risk and Monetary Policy

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Inflation: Drivers and Dynamics 2025 Conference

Macroeconomic risks and monetary policy

- Global post-pandemic events have **amplified macroeconomic risks** and have led to significant **shifts in the balance of macroeconomic risks**
  - Pre-pandemic: Contained macro risks and inflation often surprised on the downside
  - Post-pandemic: Higher likelihood of severe downturns and inflation spikes

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  - Post-pandemic: Higher likelihood of severe downturns and inflation spikes
- Yet, there is little literature on the general equilibrium effects of changes in the balance of macroeconomic risks
- We develop a tractable quantitative modeling framework to investigate:
  - How do shifts in the balance of macroeconomic risks influence the economy?
  - How should the central bank respond to changes in the balance of risks?

Model setup

New-Keynesian model with inefficient shocks featuring time-varying asymmetry

- Agents receive news about changes in the skew, shaping their perception of risks
- Evolving perceptions create time-varying belief asymmetry which affect outcomes
- Beliefs asymmetry complicates the trade-off for the central bank

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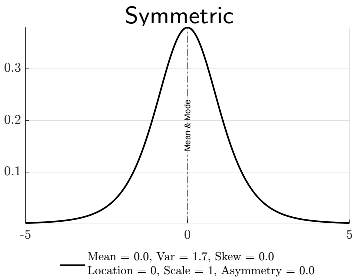
$$\Downarrow$$

Representation Theorem

Up to first-order, a model with asymmetric risks has an *equivalent* belief representation

- Analytical solution for optimal monetary policy
- Tractable risk analysis in quantitative DSGE models

Asymmetric risk: what do we mean?



In linear model, with symmetric risk, no first-order effects (certainty equivalence)

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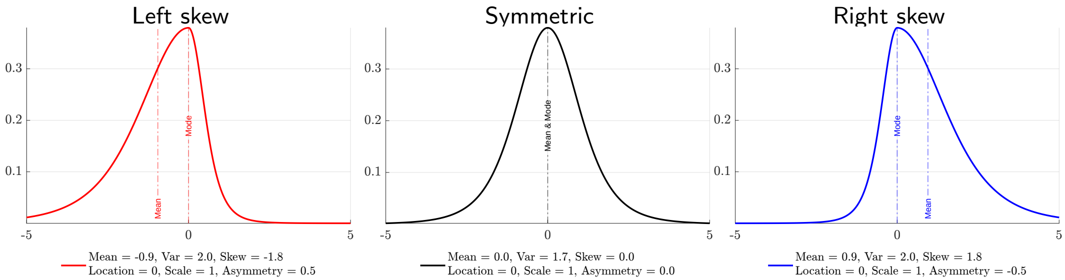

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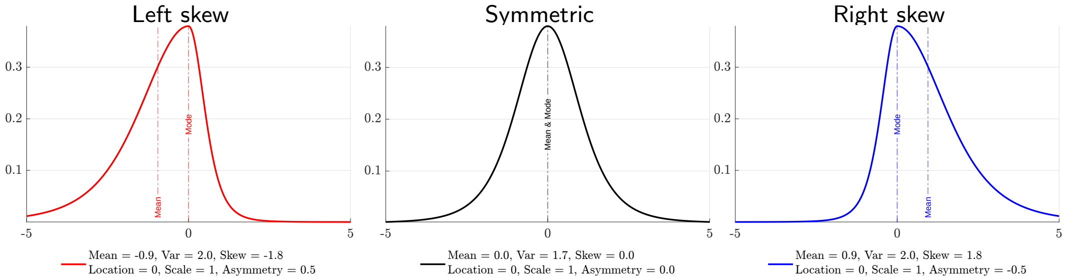

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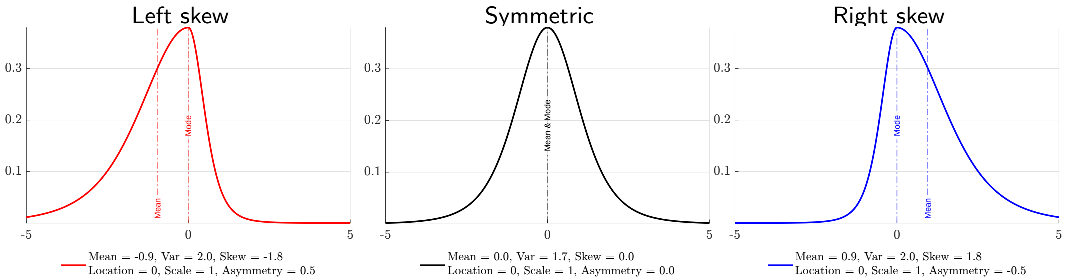
Asymmetry: drives a wedge between the mean and the mode of the distribution

$$Skew \approx \frac{Mean - Mode}{Sdev}$$

Skewness: measures the tilt in the balance of risks

Asymmetric risk: what do we mean?



In linear model, with symmetric risk, no first-order effects (certainty equivalence)

Asymmetry: drives a wedge between the mean and the mode of the distribution

$$Mean \approx Mode + Sdev \times Skew$$

Asymmetric risk affects equilibrium outcomes, even at first order!

Main contributions

1. **Optimal monetary policy with asymmetric risks**
   $\hookrightarrow$ Central bank to lean against the perceived balance of inflation risks

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2. **Tracking the balance of risks in <u>real-time</u>**
   $\hookrightarrow$ Strong evidence of time-varying balance of risks to inflation

Main contributions

1. **Optimal monetary policy with asymmetric risks**
   $\hookrightarrow$ Central bank to lean against the perceived balance of inflation risks

2. **Tracking the balance of risks in <u>real-time</u>**
   $\hookrightarrow$ Strong evidence of time-varying balance of risks to inflation

3. **Effect of asymmetric risks on the economy**
   $\hookrightarrow$ Embedding skewness in an empirical DSGE model
   $\hookrightarrow$ Counterfactual monetary policy: adjusting communication to inflation risks

Related literature

- Risk management approach to monetary policy: Dolado et al. (2004), Surico (2007), Kilian & Manganelli (2007, 2008), Cecchetti (2008), Moccero & Gnabo (2015), Evans et al. (2020)
- Flexible inflation targeting: Svensson (1997), Clarida et al. (1999), Giannoni & Woodford (2004), Clarida (2020)
- (Asymmetric) policy rules: Söderström (2002), Woodford (2003), Dolado et al. (2004, 2005), Galí (2008), Evans et al. (2020), Bianchi et al. (2021)
- Inflation forecasting & tail risks: Cogley & Sargent (2005), Stock & Watson (2007), Faust & Wright (2013), Manzan & Zerom (2013, 2015), Andrade et al. (2014), Adams et al. (2021), Hilscher et al. (2022), López-Salido & Loria (2024), Le Bihan et al. (2024)

1. Introduction

2. Macroeconomic Effects of Shift in Risks: Belief Representation

3. Balance of risks in real-time

4. Structural policy analysis

5. Conclusions

The New Keynesian setting

Consider standard the New-Keynesian Phillips Curve

$$\hat{\pi}_t = \kappa \hat{x}_t + \beta E_t \hat{\pi}_{t+1} + u_t, \quad u_t \sim \mathcal{F}(\mu, \sigma, \varrho_{u,t})$$

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1. Agents expect no shocks in their central scenario; e.g., $\mu = 0$ (can be relaxed)

2. $\mathcal{F}(\mu, \sigma, \varrho_{u,t})$ is a two-piece distribution with time-varying asymmetry, such that

$$E_t(u_{t+j}) = \varkappa \sigma \varrho_{u,t+j}, \quad \varkappa > 0$$

▸ Two-pieces distributions

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▸ Two-pieces distributions

❸ Agents receive noisy signals about the evolving asymmetry of $\mathcal{F}_{t+j}$

$$s_t^j = \varrho_{u,t+j} + \eta_t^j, \quad \forall j \in \{1, \ldots, J\}$$

The beliefs representation

Cost-push shocks defined as a sequence of dummy surprise and anticipated shocks

$$u_t \equiv \sum_{j=0}^{J} \varphi_{t-j}^{j}.$$

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Up to the first-order, a model with **(time-varying) asymmetric risks** can be represented as one with symmetric, zero-mean shocks, augmented by additional **beliefs shocks**.

▸ General Setting

Optimal monetary policy in the baseline NK model

$$\max_{\hat{x}_t, \hat{\pi}_t} \quad \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t^2 + \alpha_x \hat{x}_t^2 \right)$$

$$\text{s.t.} \quad \hat{x}_t = E_t \hat{x}_{t+1} - \varsigma^{-1} \left(\hat{i}_t - E_t \hat{\pi}_{t+1} \right), \tag{IS}$$

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❶ Skew in the shock distribution affects expectations of endogenous variables:

$$E_t\hat{\pi}_{t+1} = \vartheta_\pi\varrho_{u,t}\neq 0, \qquad E_t\hat{x}_{t+1} = \vartheta_x\varrho_{u,t}\neq 0$$

▸ Solution

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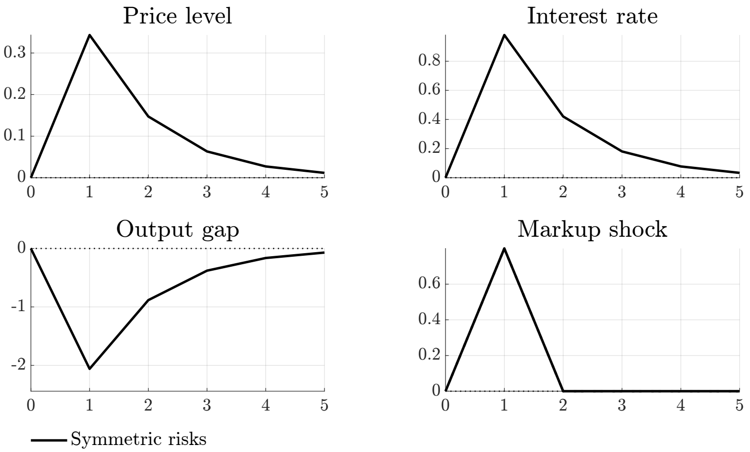
2. Belief representation ($J = 1$):

$$u_t = \varepsilon_{u,t} + \varphi_t^0 + \varphi_{t-1}^1, \qquad \varepsilon_{u,t} \sim N_t(0, \sigma_u)$$

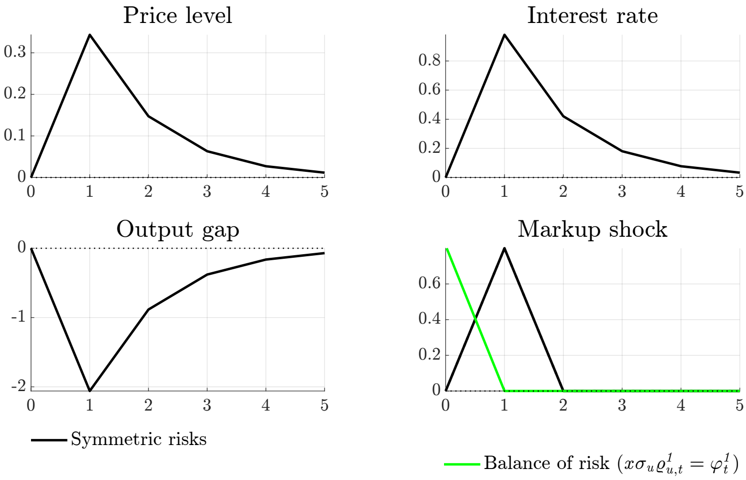
▸ Solution

Optimal Response to a Cost-push shock (in period 1)



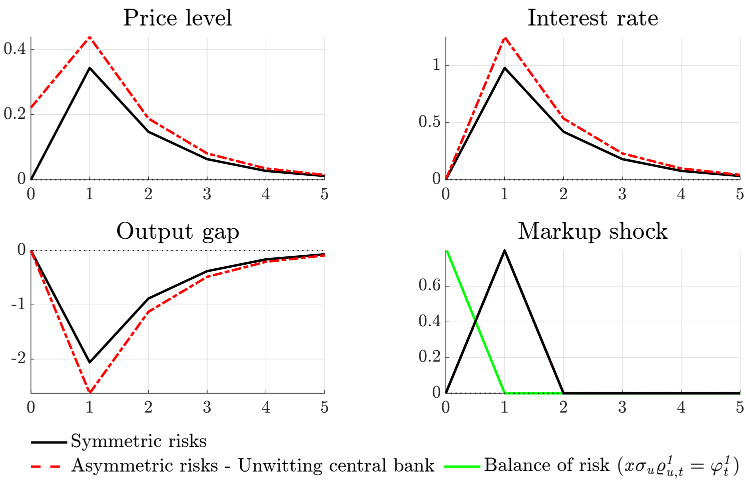
Cost-push shock lead to a tradeoff in the (optimal) MP response

Response to (anticipated) upside inflation risk



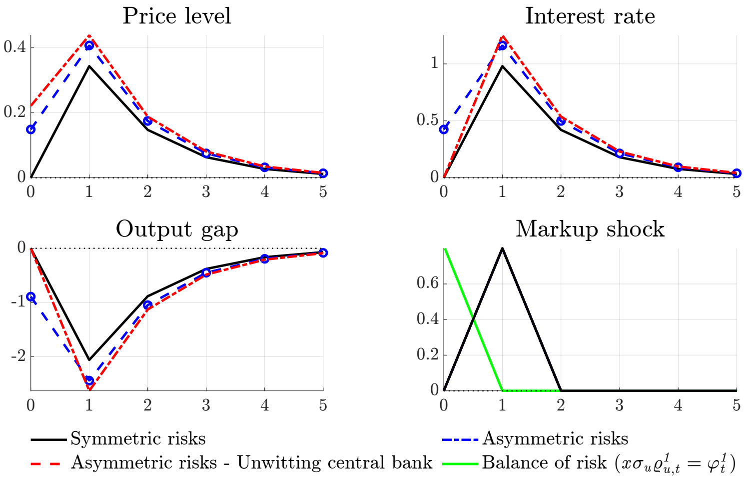
Agents expect upside risks one period ahead.

Response to (anticipated) upside inflation risk



What if the central bank is unaware of the balance of risks?

Response to (anticipated) upside inflation risk



Optimal monetary policy response: lean against inflation risk!

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Tracking Time-varying Risk

We estimate shifts in the balance of QoQ US core PCE inflation risks in *real-time*

$$\pi_t \sim skt_\nu(\mu_t, \sigma_t, \varrho_t)$$

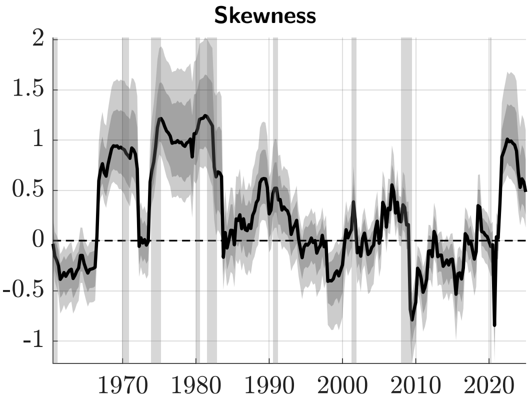
μ_t: location (central scenario)

σ_t: scale

ϱ_t: asymmetry

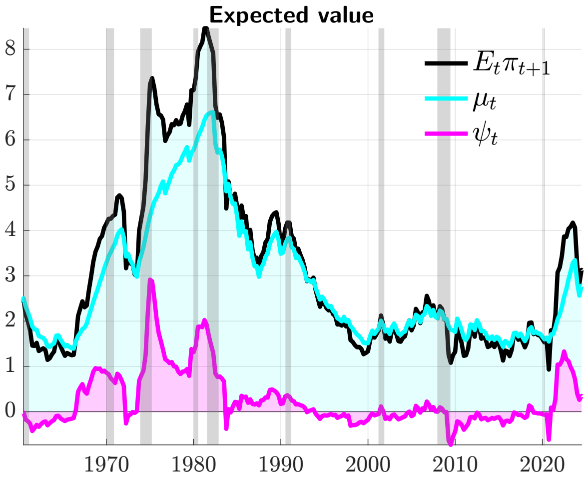
- Permanent and transitory decomposition of parameters ▸ Specification
- *Score driven* approach to parameters' time variation (as in Delle Monache, De Polis & Petrella 2024) ▸ Updates ▸ Estimation ▸ MC Simulations
- Akin to an extension of Stock & Watson (2007) with time-varying skewness

Time-varying moments: skewness



- Mean and Volatility in line with UCSV estimates ▸ Mean and Vol
- Regime-like dynamics, similar to high- and low-inflation states.
- Recent skew have greatly reduced, but still persistently above 0.

Time-varying moments: Mean decomposition



$$E_t\pi_{t+h} = \mu_{t+h} + \underbrace{g(\nu)\sigma_{t+h}\varrho_{t+h}}_{\psi_{t+h}}$$

- Upside risks in 1970-80s and post-pandemic
- Not much downside risk post GFG

$$\Downarrow$$

negative skew but low vol

Real-time predictive accuracy & robustness

- Significant out-of-sample gains in real-time
  - substantial gains across all horizons compared to Stock & Watson (2007) ▸ Table
  - omitting skewness deteriorates accuracy ▸ Table
  - event forecast marginally improves upon SPF's annual predictions. ▸ Table
- Pattern of time-varying skewness similar for
  - inflation at different frequencies, e.g. yoy inflation ▸ skew π_t^{yoy}
  - different measures of inflation, e.g. PCE, CPI, Core CPI, PCE defl. ▸ Other measures
- Model's skewness anticipates model-free measures of asymmetry
  - data-based conditional skew, e.g. rolling quantile skew or sample skew ▸ Comparison

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Skewed risk in a standard DSGE model

We augment Smets & Wouters (2007) with a belief representation:

$$\hat{\pi}_t = \kappa_1 \hat{\pi}_{t-1} + \kappa_2 \hat{mc}_t + \kappa_3 E_t \hat{\pi}_{t+1} + \sum_{j=0}^{J} \varphi_{t-j}^{j}.$$

Anticipated shocks calibrated to match the *revisions in the balance of inflation risks* extracted from real-time data

▸ Details

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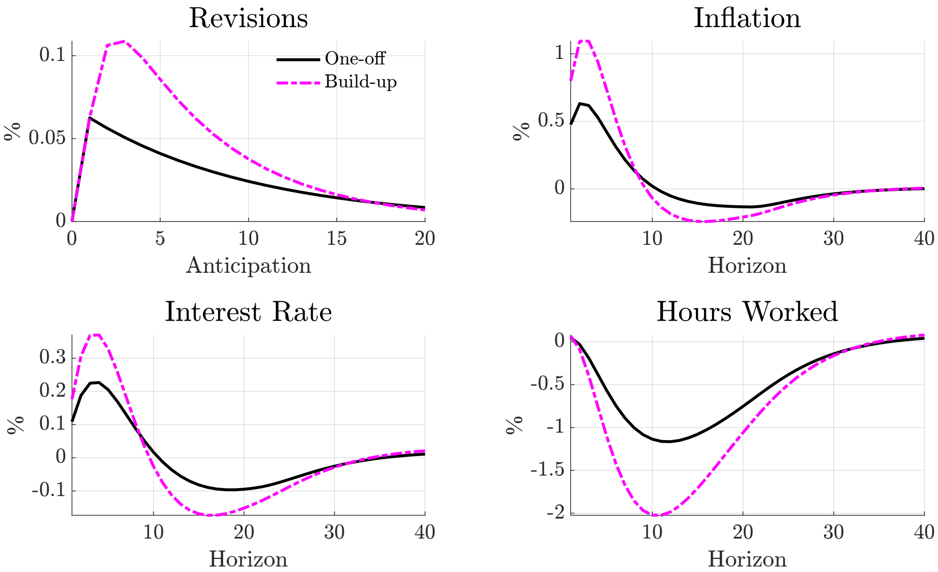
The model can be solved using standard techniques and has a solution

$$\boldsymbol{s}_t = \Gamma \boldsymbol{s}_{t-1} + \Omega \boldsymbol{e}_t,$$

where $\boldsymbol{e}_t$ includes all shocks $\boldsymbol{\varepsilon}_t$ and the dummy shocks $\{\varphi_t^j\}_{j=0}^J$

Macroeconomic effects of shifts in the balance of risk



Risk-adjusted inflation targeting (RAIT)

Central bank commits to a path of $\hat{\pi}^{\star}_{t+j|t}$ to offset the effect of asymmetric risks

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1-\rho_i)\left[\phi_x \hat{x}_t + \phi_\pi \left(\hat{\pi}_t - \sum_{j=1}^{J} \hat{\pi}^{\star}_{t|t-j}\right)\right]$$

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Forward guidance shocks

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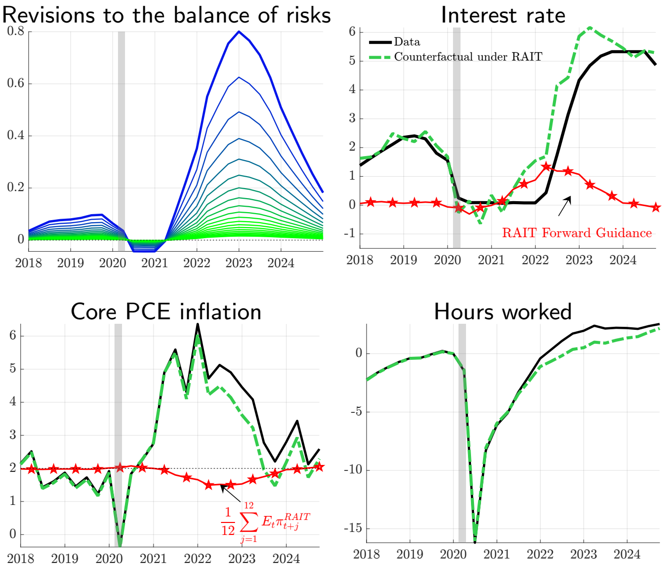
- Forward guidance as announced tilts to inflation target
- $\{\hat{\pi}^{\star}_{t+j|t}\}_{j=0}^{J}$ chosen to offset skewed risk

$$E\left[\pi_{t+h} \mid \{\varphi_t^j\}_{j=1}^{J}\right] + E\left[\pi_{t+h} \mid \{\pi^{\star}_{t+j|t}\}_{j=1}^{J}\right] = 0$$

▸ Details

Counterfactual under RAIT



Euro Area

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Conclusions

- Asymmetric inflation risks: strong evidence in the data
- Beliefs representation: *tractable* framework for time-varying macroeconomic risks
- Optimal policy: lean against the perceived balance of risks ▸ SEP 2024
- Macroeconomic response: revisions to balance of risks affect the economy
- Risk-Adjusted Inflation Targeting: communications about future interest rates influenced by the balance of risks

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The Model Environment

$$\mathbf{A}_0\mathbf{z}_t = \mathbf{A}_f E_t\mathbf{z}_{t+1} + \mathbf{A}_b\mathbf{z}_{t-1} + \mathbf{B}_s\boldsymbol{\epsilon}_t^s + \mathbf{b}_a\epsilon_t^a. \tag{1}$$

The vectors of i.i.d. disturbances includes:

- all the symmetric shocks (e.g., normally distributed): $\boldsymbol{\epsilon}_t^s$
- an *asymmetric* shock: ϵ_t^a, with distribution f$(\mu_a, \sigma_a, \varrho_a)$, where μ_a denotes the mode, σ_a the scale, and ϱ_a an asymmetry parameter that measures the degree of skewness.

back

Model Solution

The solution of the linear rational expectations model with time-varying skewness is:

$$\mathbf{z}_t = \mathbf{\Theta}_1 \mathbf{z}_{t-1} + \mathbf{\Theta}_0 \left[\boldsymbol{\epsilon}_t^s \ \epsilon_t^a \right]' + \mathbf{\Theta}_y \sum_{j=1}^{\infty} \mathbf{\Theta}_f^{j-1} \mathbf{\Theta}_z \mathbf{\Xi} E_t \epsilon_{t+j}^a, \tag{2}$$

where the matrices $\mathbf{\Theta}_0$, $\mathbf{\Theta}_1$, $\mathbf{\Theta}_y$, and $\mathbf{\Theta}_z$ are functions of $\mathbf{A}_0$, $\mathbf{A}_b$, $\mathbf{A}_f$, $\mathbf{B}_s$, and $\mathbf{b}_a$, and $\mathbf{\Xi}$ is a selection vector.

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We assume (a) agents expect no shocks in their central scenario: $\mu_a = 0$; (b) that asymmetric risk can be represented as a two-piece asymmetric distribution with time varying asymmetry $\rightarrow$ $E_t(\epsilon_{t+j}^a) = \varkappa \sigma_a \varrho_{a,t+j}$

back

Asymmetric risk and macroeconomic outcomes

- Rational agents solve the signal extraction problem leading to the following update in expectations about the asymmetry parameter $j \in \{1, ..., J\}$ periods ahead: $\varrho^j_{a,t} \equiv E_t \varrho_{a,t+j} - E_{t-1} \varrho_{a,t+j}$.

- Revisions in risk affect the macroeconomy:

$$\frac{\partial \mathbf{z}_t}{\partial \varrho^j_{a,t}} = \boldsymbol{\Theta}_y \boldsymbol{\Theta}_f^{j-1} \boldsymbol{\Theta}_z \boldsymbol{\Xi} \varkappa \sigma_a, \tag{3}$$

back

Belief representation

We show that, up to a first-order approximation, a model with **(time-varying) asymmetric risks** can be equivalently represented as one with symmetric, zero-mean shocks, augmented by additional **beliefs shocks**.

- Define the asymmetric shock as: $\epsilon_t^a \equiv \sum_{j=0}^{J} \varphi_{t-j}^j$, where φ_t^j denotes the dummy beliefs shocks.
- The equilibrium dynamics of the beliefs representation are identical to the ones of the actual economy if:

1. $\varphi_t^j = E_t \epsilon_{a,t+j} - E_{t-1} \epsilon_{a,t+j} = \varkappa \sigma_a \left(E_t \varrho_{a,t+j} - E_{t-1} \varrho_{a,t+j} \right) = \varkappa \sigma_a \varrho_{a,t}^j$
2. $\varphi_t^0 = \epsilon_{a,t} - \sum_{j=1}^{J} \varphi_{t-j}^j$ (i.e. dummy shocks are belief shocks, they never realize)

back

Definition of a Unimodal Two-Piece Distribution

A **unimodal two-piece distribution** is a probability density function (PDF) that:

- Has a single peak (unimodal) at a central point (e.g., mode or median).
- Is defined by **two different functions** on either side of the peak.
- Can be **symmetric or asymmetric**, depending on the parameterization.

General form:

$$f(x) = \begin{cases} A_L g_L(x), & x < c \\ A_R g_R(x), & x \geq c \end{cases}$$

where c is the splitting point, and A_L, A_R are normalization constants to ensure the total probability integrates to 1.

Back

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- Can be **symmetric or asymmetric**, depending on the parameterization.

Two-Piece Normal Distribution:

$$f(x) = \begin{cases} \frac{2\sigma_L}{\sigma_L+\sigma_R}\phi\left(\frac{x-c}{\sigma_L}\right), & x < c \\ \frac{2\sigma_R}{\sigma_L+\sigma_R}\phi\left(\frac{x-c}{\sigma_R}\right), & x \geq c \end{cases}$$

when $\sigma_L = \sigma_R$ we recover the symmetric Normal distribution.

Back

Optimal policy with asymmetric risks

$$
\begin{aligned}
& \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t^2 + \alpha_x \hat{x}_t^2 \right) \\
s.t. \quad & \hat{x}_t = E_t \hat{x}_{t+1} - \varsigma^{-1} \left(\hat{i}_t - E_t \hat{\pi}_{t+1} \right) \\
& \hat{\pi}_t = \kappa \hat{x}_t + \beta E_t \hat{\pi}_{t+1} + u_t, \\
& u_t = \varepsilon_{u,t},
\end{aligned}
$$

Optimal policy with asymmetric risks

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\hat{\pi}_t^2+\alpha_x\hat{x}_t^2\right)$$

$$\begin{aligned} s.t. \quad \hat{x}_t &= E_t\hat{x}_{t+1}-\varsigma^{-1}\left(\hat{i}_t-E_t\hat{\pi}_{t+1}\right) \\ \hat{\pi}_t &= \kappa\hat{x}_t+\beta E_t\hat{\pi}_{t+1}+u_t, \\ u_t &= \varepsilon_{u,t}, \qquad \varepsilon_{u,t}\sim N(0,\sigma_u) \end{aligned}$$

Solution:

$$E_t\bar{p}_{t+1}=\eta\bar{p}_t, \quad E_t\varepsilon_{u,t+1}=0$$

and the optimal policy rule

$$\hat{i}_t=-(1-\eta)\left[1-\sigma\frac{\kappa}{\alpha_x}\right]\bar{p}_t,$$

such that $\hat{x}_t=\eta\hat{x}_{t-1}-\frac{\kappa}{\alpha_x}\lambda u_t$.

Back

Optimal policy with asymmetric risks

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\hat{\pi}_t^2+\alpha_x\hat{x}_t^2\right)$$

$$\begin{aligned} s.t. \quad \hat{x}_t &= E_t\hat{x}_{t+1}-\varsigma^{-1}\left(\hat{i}_t-E_t\hat{\pi}_{t+1}\right) \\ \hat{\pi}_t &= \kappa\hat{x}_t+\beta E_t\hat{\pi}_{t+1}+u_t, \\ u_t &= \varepsilon_{u,t}, \qquad \varepsilon_{u,t}\sim\mathcal{F}_t(0,\sigma_u,\varrho_u,\ldots) \end{aligned}$$

Solution:

$$E_t\bar{p}_{t+1}\neq\eta\bar{p}_t, \quad E_t\varepsilon_{u,t+1}\neq 0$$

and the optimal policy rule

$$\hat{i}_t\neq-(1-\eta)\left[1-\sigma\frac{\kappa}{\alpha_x}\right]\bar{p}_t,$$

such that $\hat{x}_t\neq\eta\hat{x}_{t-1}-\frac{\kappa}{\alpha_x}\lambda u_t$.

Back

Optimal policy with asymmetric risks

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\hat{\pi}_t^2+\alpha_x\hat{x}_t^2\right)$$

$$s.t.\quad \hat{x}_t = E_t\hat{x}_{t+1}-\varsigma^{-1}\left(\hat{i}_t - E_t\hat{\pi}_{t+1}\right)$$

$$\hat{\pi}_t = \kappa\hat{x}_t+\beta E_t\hat{\pi}_{t+1}+u_t,$$

$$u_t = \varepsilon_{u,t}+\varphi_t^0+\varphi_{t-1}^1,\qquad \varepsilon_{u,t}\sim N(0,\sigma_u)$$

Solution (belief representation):

$$E_t\bar{p}_{t+1}=\eta\bar{p}_t+\zeta E_t\varphi_{t+1}^1,\quad E_t\varepsilon_{u,t+1}=0$$

and the optimal policy rule

$$\hat{i}_t = -(1-\eta)\left[1-\sigma\frac{\kappa}{\alpha_x}\right]\left(\bar{p}_t+\lambda\varphi_t^1\right),$$

such that $\hat{x}_t=\eta\hat{x}_{t-1}-\frac{\kappa}{\alpha_x}\left(\lambda u_t+\zeta\varphi_t^1\right)$.

Back

Model: specification

$$\pi_t \sim skt_\nu(\mu_t, \sigma_t, \varrho_t)$$

μ_t: location (central scenario)

σ_t: scale

ϱ_t: asymmetry

Model: specification

$$\pi_t \sim skt_\nu(\mu_t, \sigma_t, \varrho_t)$$

μ_t: location (central scenario)

σ_t: scale

ϱ_t: asymmetry

Let $f_t = (\mu_t, \log\sigma_t, \operatorname{atanh}\varrho_t)'$, then $f_{i,t+1} = \bar{f}_{i,t+1} + \tilde{f}_{i,t+1}$:

$$\bar{f}_{i,t+1} = \bar{f}_{i,t} + a_i s_{i,t} \qquad \text{(Permanent)}$$

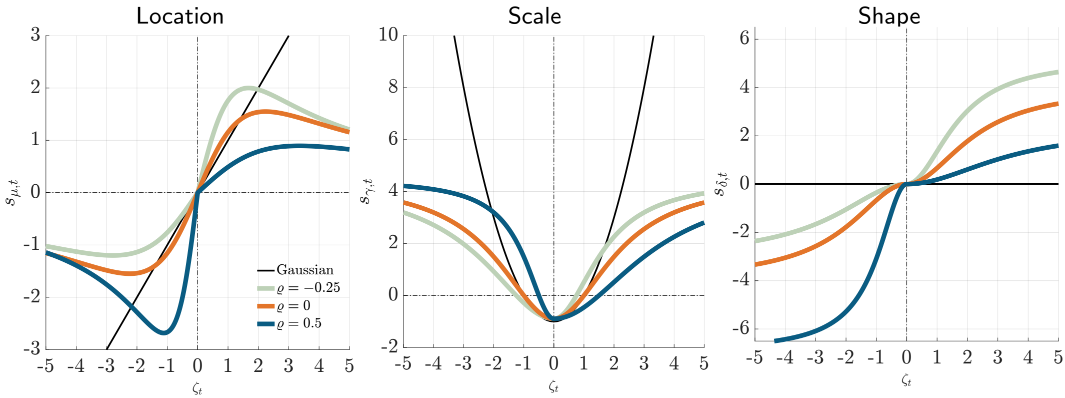
$$\tilde{f}_{i,t+1} = \phi_i \tilde{f}_t + b_i s_{i,t} \qquad \text{(Transitory)}$$

where $s_t = \mathbf{S}_{t-1}\nabla_t$, $\nabla_t = \frac{\partial \ell_t}{\partial f_t}$, and $\mathbf{S}_{t-1} = \mathcal{I}_{t-1}^{-1} = \mathbb{E}_{t-1}\left[\frac{\partial \ell_t}{\partial f_t \partial f_t'}\right]^{-1}$.

- s_t maps ε_t into an appropriate update for f_t (Creal et al. 2013, Harvey 2013)
- The model tracks skewness only when it is present (Delle Monache et al. 2024)

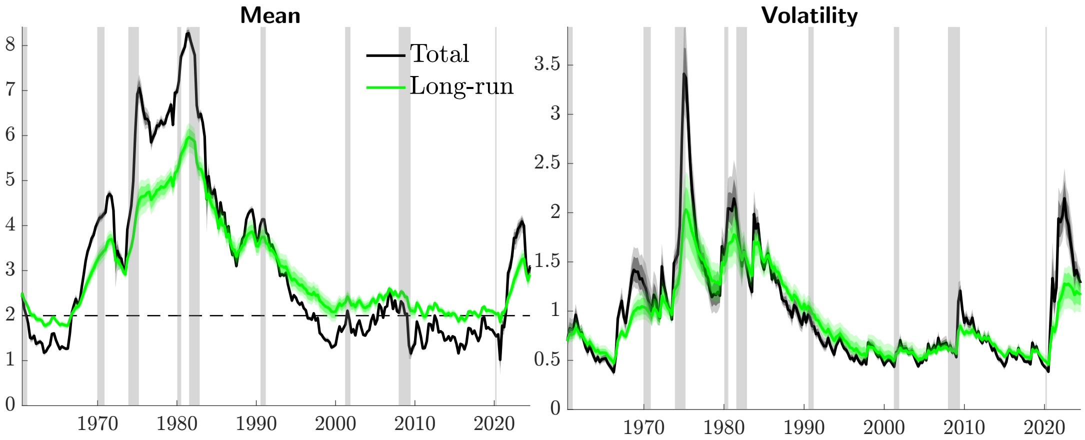
Back

Moments updating



◂ Back

Time-varying moments: mean & volatility



- Post GFC, *deflationary bias* around persistent component
- Low and stable volatility since mid '80s

Back

Comparison with UCSV

	MSFE	CRPS	CRPS Decomposition			Event Forecasts		
			Right	Left	Center	$\pi_{t+h} < 1.5$	$1.5 \leq \pi_{t+h} \geq 2.5$	$\pi_{t+h} > 2.5$
h = 1	**0.969**	0.995	0.998	0.992	0.995	**0.956**	**0.966**	**0.967**
	(0.011)	(0.333)	(0.424)	(0.186)	(0.364)	(0.001)	(0.004)	(0.001)
h = 4	**0.925**	**0.958**	**0.979**	**0.940**	**0.954**	0.981	0.981	**0.987**
	(0.000)	(0.001)	(0.052)	(0.000)	(0.000)	(0.107)	(0.115)	(0.064)
h = 8	**0.884**	**0.927**	**0.939**	**0.921**	**0.920**	**0.975**	**0.970**	1.007
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.074)	(0.014)	(0.702)

Back

Is skewness improving predictions?

	h = 1	h = 2	h = 3	h = 4	h = 8
MSFE	**0.865**	**0.901**	**0.926**	**0.970**	1.006
	(0.000)	(0.000)	(0.000)	(0.000)	(0.939)
CRPS	**0.957**	**0.970**	**0.969**	**0.980**	**0.995**
	(0.001)	(0.000)	(0.000)	(0.006)	(0.066)
	CRPS decomposition				
Right	**0.945**	**0.945**	**0.952**	**0.966**	**0.985**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)
Left	**0.966**	0.991	**0.983**	0.993	1.004
	(0.000)	(0.142)	(0.006)	(0.146)	(0.913)
Center	**0.959**	**0.975**	**0.972**	**0.984**	0.997
	(0.001)	(0.008)	(0.000)	(0.007)	(0.147)

Results are reported as the ratio of the Skt model over a specification without skewness.

Back

Comparison with SPF (event forecasts)

We compare the end-of-year event forecasts by means of the Brier score

$$b_t = \frac{1}{N}\sum_{n=1}^{N}(p_t - o_t)^2$$

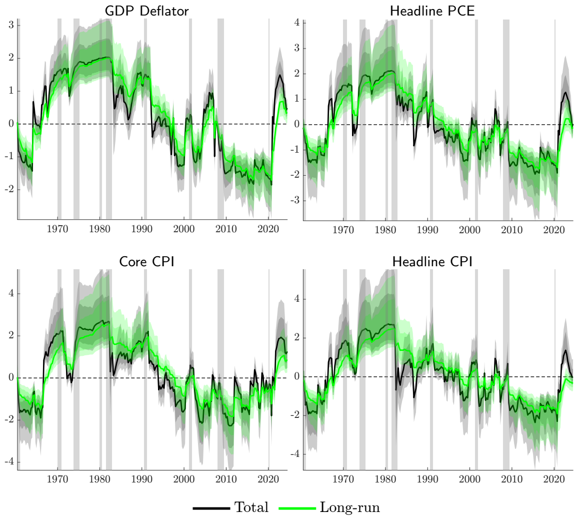
where p_t is the predicted probability of the event, and o_t is a boolean outcome indicator

	$\pi_t^{Q4} < 1.5\%$				$1.5\% \leq \pi_t^{Q4} \leq 2.5\%$				$\pi_t^{Q4} > 2.5\%$			
	h = 1	h = 2	h = 3	h = 4	h = 1	h = 2	h = 3	h = 4	h = 1	h = 2	h = 3	h = 4
SPF	1.216	0.929	1.217	0.890	1.118	0.928	1.376	1.120	0.910	0.235	0.530	0.847
UCSV	0.998	1.063	1.023	0.995	1.053	1.033	1.031	0.984	0.917	0.738	0.831	0.838

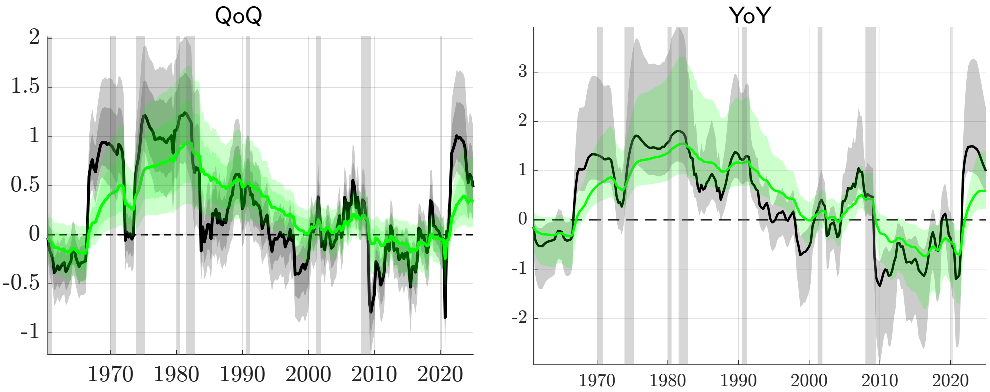
Results are reported as the ratio of the Skt model over the SPF's.

◂ Back

Alternative Measures of Inflation: Skewness



YoY Inflation



Back

Aligning the beliefs representation to real-time estimates of risks

Set surprise and anticipated dummy shocks, $\{\varphi^j\}_{j=0}^J$, as follows:

$$\begin{bmatrix} \varepsilon_t^p - \sum_{j=1}^J \varphi_{t-j}^j \\ \psi_{t+1|t} - \psi_{t+1|t-1} \\ \vdots \\ \psi_{t+J|t} - \psi_{t+J|t-1} \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{0}_{1\times J} \\ \Omega^S & \Omega^N \end{bmatrix} \begin{bmatrix} \varphi_t^0 \\ \varphi_t^1 \\ \vdots \\ \varphi_t^J \end{bmatrix},$$

Ω^S and Ω^N: Impact matrices of the dummy shocks on inflation expectations

1. Ensure the realization of shocks is the same in the model and its representation

Back

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Ω^S and Ω^N: Impact matrices of the dummy shocks on inflation expectations

1. Ensure the realization of shocks is the same in the model and its representation
2. Set dummy surprise and news shocks to match the bias on inflation expectations, $\psi_{t+h|t}$ due to the balance of risks estimated in real time

Back

RAIT: implementation

$$E\left[\pi_{t+h} \mid \{\varphi_t^j\}_{j=1}^J\right] + E\left[\pi_{t+h} \mid \{\pi^\star_{t+j|t}\}_{j=1}^J\right] = 0$$

RAIT: implementation

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$$\Downarrow$$

$$-\begin{bmatrix} \psi_{t+1|t} - \psi_{t+1|t-1} \\ \psi_{t+2|t} - \psi_{t+2|t-1} \\ \vdots \\ \psi_{t+J|t} - \psi_{t+J|t-1} \end{bmatrix} = \Omega_{FG} \begin{bmatrix} \hat{\pi}^\star_{t+1|t} \\ \hat{\pi}^\star_{t+2|t} \\ \vdots \\ \hat{\pi}^\star_{t+J|t} \end{bmatrix}$$

where Ω_{FG} is the impact matrix of FG shocks on inflation expectations

Back

RAIT: implementation

$$E\left[\pi_{t+h} \mid \{\varphi_t^j\}_{j=1}^J\right] + E\left[\pi_{t+h} \mid \{\pi^\star_{t+j|t}\}_{j=1}^J\right] = 0$$

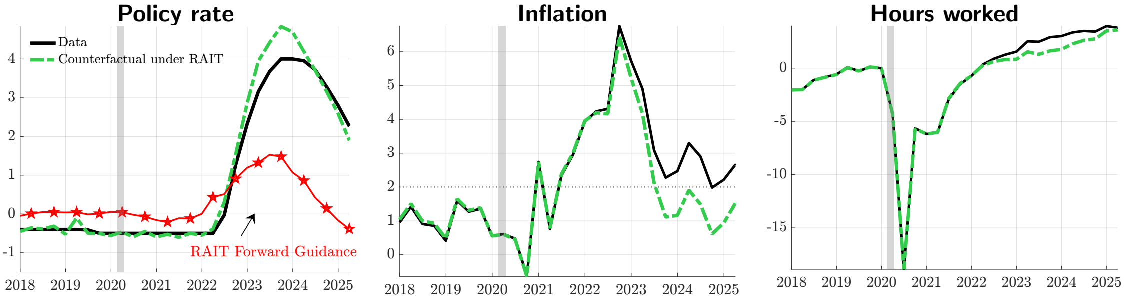
$$\Downarrow$$

$$\begin{bmatrix} \hat{\pi}^\star_{t+1|t} \\ \hat{\pi}^\star_{t+2|t} \\ \vdots \\ \hat{\pi}^\star_{t+J|t} \end{bmatrix} = -\Omega_{FG}^{-1} \begin{bmatrix} \psi_{t+1|t} - \psi_{t+1|t-1} \\ \psi_{t+2|t} - \psi_{t+2|t-1} \\ \vdots \\ \psi_{t+J|t} - \psi_{t+J|t-1} \end{bmatrix}$$

Pin down FG shocks, communicating asymmetric response to inflation

Back

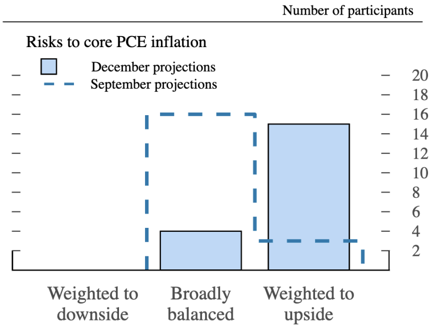
The Euro Area



◂ US

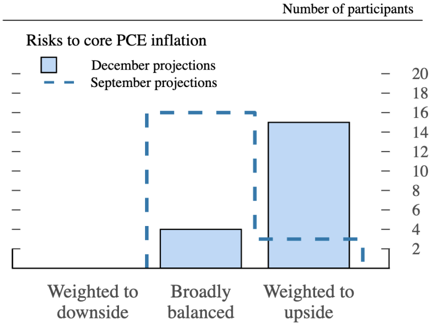
Asymmetric risks: FOMC Dec 2024

- Increased perception of upside risks to inflation (tariffs?)



Asymmetric risks: FOMC Dec 2024

- Increased perception of upside risks to inflation (tariffs?)
- Lead to anticipating tighter stance



$\Rightarrow$

Percent

Variable	Median[1] 2024	2025	2026	2027	Longer run
Change in real GDP	2.5	2.1	2.0	1.9	1.8
September projection	2.0	2.0	2.0	2.0	1.8
Unemployment rate	4.2	4.3	4.3	4.3	4.2
September projection	4.4	4.4	4.3	4.2	4.2
PCE inflation	2.4	2.5	2.1	2.0	2.0
September projection	2.3	2.1	2.0	2.0	2.0
Core PCE inflation[4]	2.8	2.5	2.2	2.0	
September projection	2.6	2.2	2.0	2.0	
Memo: Projected appropriate policy path					
Federal funds rate	4.4	3.9	3.4	3.1	3.0
September projection	4.4	3.4	2.9	2.9	2.9

Asymmetric risks: FOMC Dec 2024

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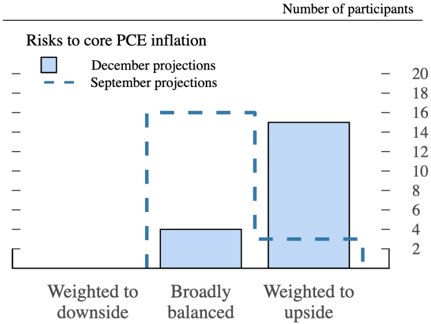

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"The Committee will assess incoming data, the evolving outlook, and the balance of risks. We're not on any preset course." FOMC, 18/12/2024.

Back

Summary of Economic Projections, 12/2024

Percent

Variable	Median[1]					Central Tendency[2]					Range[3]				
	2024	2025	2026	2027	Longer run	2024	2025	2026	2027	Longer run	2024	2025	2026	2027	Longer run
Change in real GDP	2.5	2.1	2.0	1.9	1.8	2.4–2.5	1.8–2.2	1.9–2.1	1.8–2.0	1.7–2.0	2.3–2.7	1.6–2.5	1.4–2.5	1.5–2.5	1.7–2.5
September projection	2.0	2.0	2.0	2.0	1.8	1.9–2.1	1.8–2.2	1.9–2.3	1.8–2.1	1.7–2.0	1.8–2.6	1.3–2.5	1.7–2.5	1.7–2.5	1.7–2.5
Unemployment rate	4.2	4.3	4.3	4.3	4.2	4.2	4.2–4.5	4.1–4.4	4.0–4.4	3.9–4.3	4.2	4.2–4.5	3.9–4.6	3.8–4.5	3.5–4.5
September projection	4.4	4.4	4.3	4.2	4.2	4.3–4.4	4.2–4.5	4.0–4.4	4.0–4.4	3.9–4.3	4.2–4.5	4.2–4.7	3.9–4.5	3.8–4.5	3.5–4.5
PCE inflation	2.4	2.5	2.1	2.0	2.0	2.4–2.5	2.3–2.6	2.0–2.2	2.0	2.0	2.4–2.7	2.1–2.9	2.0–2.6	2.0–2.4	2.0
September projection	2.3	2.1	2.0	2.0	2.0	2.2–2.4	2.1–2.2	2.0	2.0	2.0	2.1–2.7	2.1–2.4	2.0–2.2	2.0–2.1	2.0
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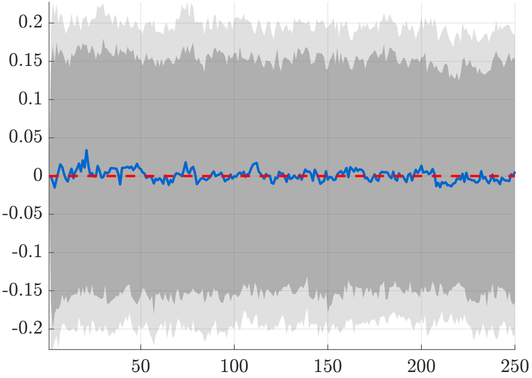
September projection	2.6	2.2	2.0	2.0		2.6–2.7	2.1–2.3	2.0	2.0		2.4–2.9	2.1–2.5	2.0–2.2	2.0–2.2	
Memo: Projected appropriate policy path															
Federal funds rate	4.4	3.9	3.4	3.1	3.0	4.4–4.6	3.6–4.1	3.1–3.6	2.9–3.6	2.8–3.6	4.4–4.6	3.1–4.4	2.4–3.9	2.4–3.9	2.4–3.9
September projection	4.4	3.4	2.9	2.9	2.9	4.4–4.6	3.1–3.6	2.6–3.6	2.6–3.6	2.5–3.5	4.1–4.9	2.9–4.1	2.4–3.9	2.4–3.9	2.4–3.8

Back

Simulation Exercise

Would the model find any skewness when there is none in the data?



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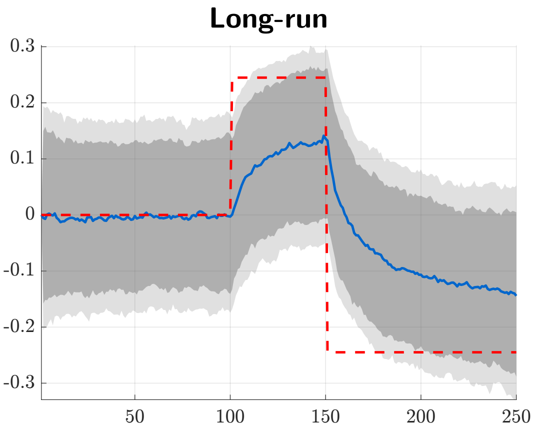
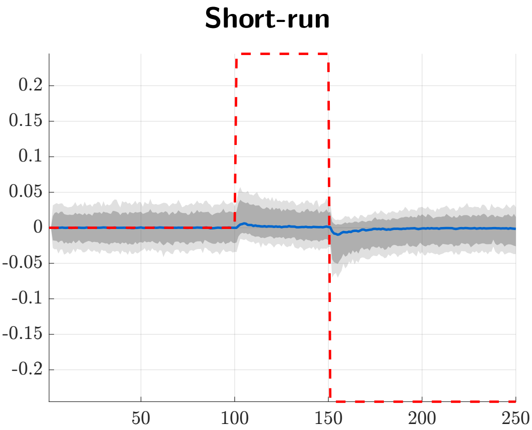
Simulation Exercise

How does the model handle sudden structural breaks?





▸ Back

Adaptive Metropolis-Hastings

Given the vector of static parameters θ:

MH steps	
Draw:	$\theta^* = \theta^{j-1} + \epsilon, \ \epsilon \sim \mathcal{N}(0, \Sigma_H)$
Accept:	$\theta^j = \theta^*$ with probability $p = \min\left[1, \frac{e^{\ell(\theta^j)}}{e^{\ell(\theta^{j-1})}}\right]$
Adaptive steps	
Rescale:	$\sigma_s = \sigma_s r(\tilde{\alpha}^s)$, every s draws
Reestimate:	$\Sigma_H = \frac{\tilde{K}}{\sqrt{H-1}}$, every U draws

where $r(\tilde{\alpha}^s)$ is an arbitrary function of the local acceptance rate $\tilde{\alpha}^s$ to target a 30% acceptance rate. We set $s = 100$, $U = 750$ and $H = 1000$.

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